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STUDY OF DIFFUSION FROM A LINE SOURCE IN A TURBULENT BOUNDARY LAYER

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Abstract—Diffusion of a scalar quantity (ammonia gas) from a steady line source within a two-dimensional turbulent boundary layer is studied. Using a long 6×6 ft square test section, the boundary-layer thickness varied from 5 to 11 in for distances of 3 to 43·5 ft downstream with air speeds from 9 to 16 ft/s. Measurements of mean concentrations of ammonia in air are reported, analysed and compared with a few measurements of heat transfer in similar conditions. Formulation of the results takes into consideration the transverse non-homogeneity of the velocity field and also divides the downstream diffusion field into four zones. Measurements of the mean velocity field and the mean concentration field permit the flux of mass in the vertical direction to be calculated through the equation of mass conservation. The use of an eddy-diffusivity coefficient to describe the processes of turbulent diffusion is discussed and it is shown that for a long distance downstream of the source, such a coefficient cannot be related to the local Eulerian variables of the velocity field.

NOMENCLATURE

Unless otherwise stated instantaneous values of any fluctuating variable p will be written as $p + p'$, where p is the mean value and p' is the fluctuating component. Time averages will be denoted by $\overline{(\ )}$, for example $\overline{p + p'} = p$.

C_{max},	maximum value of a concentration profile, ground-level concentration;
c,	concentration of the diffusing matter;
$F(\eta)$,	universal concentration function in the final zone, defined in equation (7);
$F(\xi)$,	defined in equation (14);
$f(\xi)$,	universal concentration function in the intermediate zone, defined in equation (4);
G,	flux of the diffusing matter per unit time for a unit width;
$g(\eta)$,	universal velocity function in the test section, equation (18);
k,	molecular diffusivity;
S_λ, S_δ, S_σ,	dimensionless functions associated with the description of $\overline{v'c'}$, defined in equations (14), (20) and (22);
U_{amb},	velocity of the ambient air stream;
u,	velocity in the x-direction;
v,	velocity in the y-direction;
x,	distance downstream from the source;
X',	distance downstream from origin of turbulent boundary layer;
y,	height above the boundary.

Greek symbols

β,	defined in equation (3);
δ,	boundary-layer thickness, $(u/U_{amb})(\delta) = 0{\cdot}99$;
δ_{av},	defined in Fig. 8;
ϵ,	coefficient of eddy diffusivity, $= -[(\overline{v'c'})/(\partial c/\partial y)]$;
η,	dimensionless height (y/δ);
λ,	characteristic height of the diffusing plume, $[c(\lambda)/C_{max}] = 0{\cdot}5$;
ν,	kinematic viscosity;
ξ,	dimensionless height (y/λ);
σ,	the variance of the concentration profile for homogeneous turbulence.

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INTRODUCTION

THE ABILITY to diffuse matter, heat and other contaminants is one of the basic characteristics of turbulent flow. Turbulent diffusion of matter and heat is of primary importance in industrial, chemical and atmospheric studies. Since the source of such contaminants is in many cases close to the solid boundaries, the study of diffusion in turbulent boundary-layer flows is of special interest.

The general problem in diffusion studies is to express the turbulent transport rate of transferable scalar quantities in terms of statistical functions of the turbulent motion and of the boundary conditions. It is obvious that a complete solution of the transport problem can be expected only if there is a complete knowledge of the turbulent motion. G. I. Taylor [1] has demonstrated that the characteristics of transport processes are related to the Lagrangian statistical functions of the turbulent motion. He has formulated such a relation for the simple case of homogeneous turbulence. Measurement of the Lagrangian statistical quantities is difficult and a relation between the Lagrangian and Eulerian variables is available only for highly simplified models.

In view of these difficulties, phenomenological theories based on the concept of a "mixing length" or an "eddy diffusivity" were introduced and have been used in meteorological and engineering studies. Such theories have attempted to relate the mean flux of the contaminant by turbulent fluctuations to known variables of the turbulent field at the same point. The widely used Fickian treatment of atmospheric diffusion assumes that the flux $q_i = \overline{u_i'c'}$ is proportional to the gradient of the concentration $(\partial c/\partial x_i)$; thus, the flux normal to the stream becomes $q_y = \overline{v'c'} = -\epsilon\,(\partial c/\partial y)$, where ϵ is called the coefficient of eddy diffusivity in analogy to the coefficient of molecular diffusivity. The existence of very large eddies comparable in size to the boundary-layer thickness itself does not justify such an analogy; however, a coefficient of eddy diffusivity can always be introduced as a mathematical operation, hoping that such a representation will simplify the problem. Such a construction was found successful in studies of free turbulence [2] where ϵ can be approximated by a constant. It was disappointing to find that in a boundary layer ϵ is not a constant [3]. In view of the results found in the study of diffusion in homogeneous turbulence, there was some hope that ϵ could be related theoretically or experimentally to simple turbulent quantities like $\overline{v'^2}$ or $-[(\overline{u'v'})/(\partial u/\partial y)]$ which corresponds to an eddy diffusivity for momentum transfer. The latter model was reported to be successful in a few cases of diffusion from an area source where a continuous flux of matter or heat, analogous to a wall shear stress, was emitted along the boundary [4]. In general, universal relations between ϵ and the turbulent quantities were not obtained but the use of the mathematical model has been continued since no theoretical work has yielded methods adequate for use in practical problems. The theoretical difficulties to formulate a model of the diffusion pattern have encouraged much experimental work.

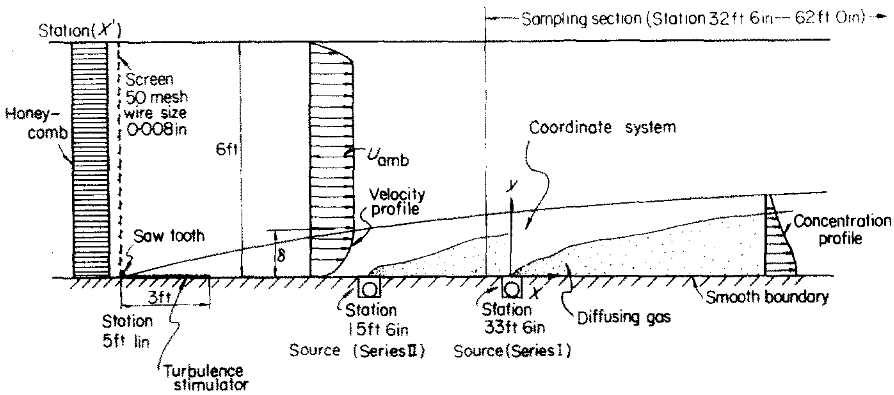

FIG. 1. Test section geometry.

Field studies of atmospheric diffusion which suffer from the inherent disadvantages associated with an uncontrolled atmosphere did not remove these difficulties. An alternative experimental approach is a wind-tunnel investigation of diffusion within boundary layers. Experimental investigations of diffusion from a source located at the solid boundary of a boundary-layer flow were reported by Wieghardt [5], Davar [6] and Malhotra [8]. Davar studied the pattern of diffusion from a point source and Malhotra investigated the effect of unstable density stratifications on the transport mechanism. Wieghardt investigated the problem of heat diffusion within a short distance downstream of ground-level line and point sources located in an otherwise isothermal boundary layer. The present paper summarizes the previous work of Poreh on diffusion from a ground-level line source, and formulates and analyses the diffusion pattern for short and large distances downstream of the source taking into consideration the non-homogeneity of the boundary layer. Wieghardt's formulation of the problem is briefly discussed and part of his data is compared with the mass-diffusion data. The experimental work which served as the basis of the analysis is discussed in the following section.

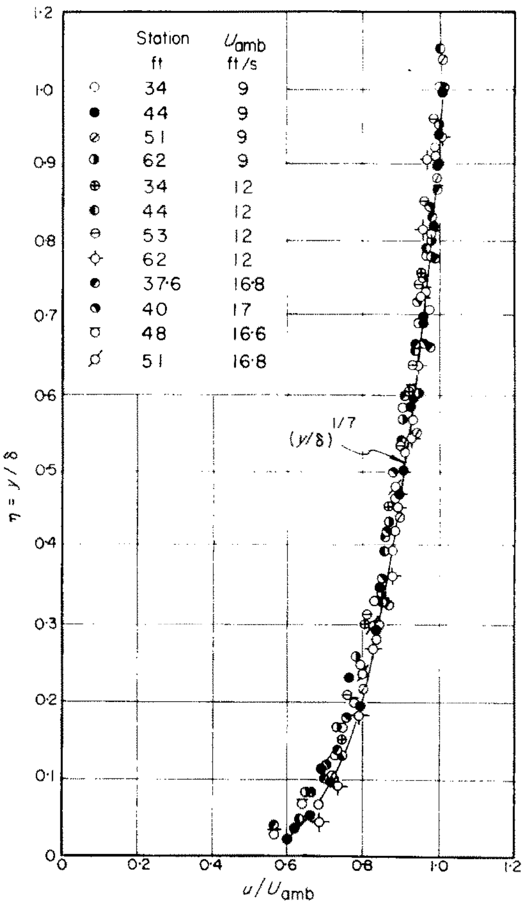

FIG. 2. Universal velocity profile.

DESCRIPTION OF THE VELOCITY FIELD—THE EXPERIMENTAL SYSTEM AND PROCEDURE

Experiments were performed in a non-circulating wind tunnel which is located in the Fluid Dynamics and Diffusion Laboratory of Colorado State University. The test section is approximately 80 ft long and 6 × 6 ft square, slightly increasing in width in the direction of the flow to provide a zero longitudinal pressure gradient.

Three ambient velocities of approximately 9, 12, and 16 ft/s were used. Mean velocities were measured by a manually balanced, constant-temperature, hot-wire anemometer. The mean-velocity profiles within the test section shown in Fig. 2 were approximately similar and the boundary-layer thickness δ varied from 5 to 11 in (Fig. 3). The boundary-layer thickness was taken to be the height at which $u = 0{\cdot}99\ U_{amb}$. The Reynolds number $U_{amb}\ (\delta/\nu)$ varied from 25 000 to 56 000. Limited measurements of the turbulence shown in Fig. 4 were taken with a constant-temperature anemometer.

Anhydrous ammonia gas (NH_3) was emitted from a line source located at ground level. The molecular diffusivity of ammonia in air at 25°C is 0·236 making the Schmidt number (ν/k) of the system approximately 0·72. Sampling rates were adjusted to approximately the velocity of the air stream except, of course, near the boundary. The minimum sampling time was one minute, but the usual sampling time was between 2 and 3 min. The sampled air–gas mixture was passed through an absorption tube containing dilute hydrochloric acid which absorbed the ammonia. The absorbed solution of ammonia was then chemically treated. Absolute quantities of ammonia were determined with a photo-electric

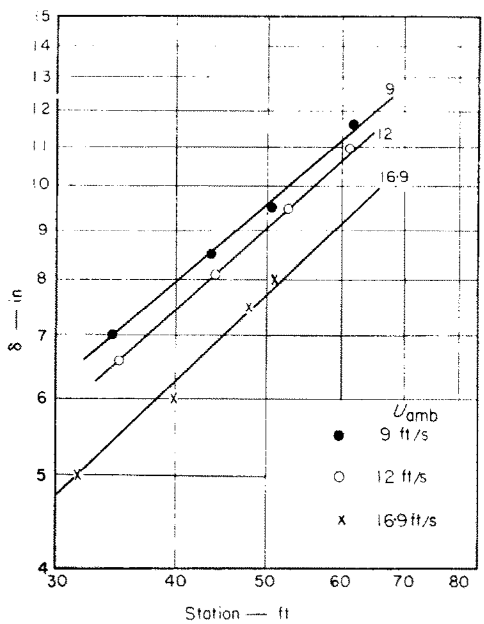

FIG. 3. Variation of the boundary-layer thickness.

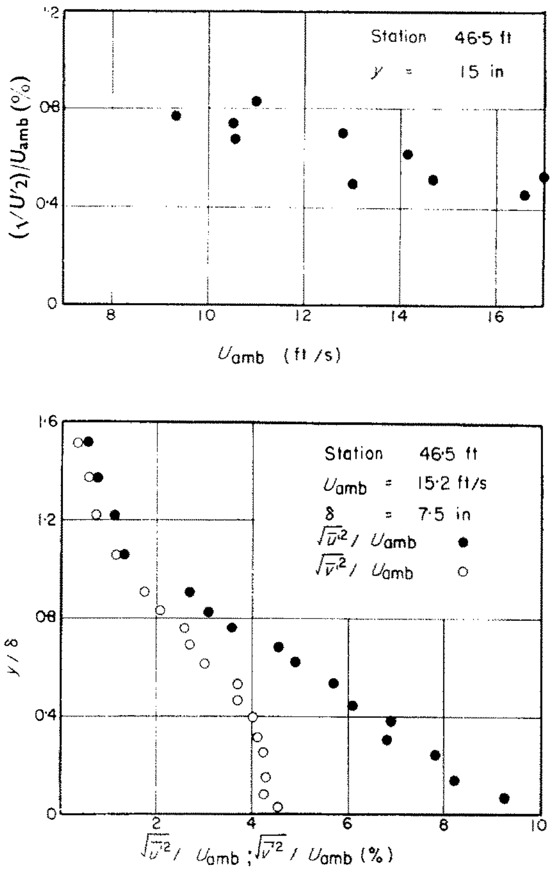

FIG. 4. Turbulence measurements.

colorimeter. Detailed description of the equipment is given in [7].

The large number of samples taken required the chemical analysis to be simple and quick. This resulted in deviation of up to 6 per cent, between separate readings of standard solutions taken in different times using different preparations of Nessler's Reagent. The colorimetric method was not accurate where very mild concentrations were involved, which influenced the recorded concentrations near the upper edge of the plume.

In spite of the above limitations the data were reproducible within a deviation of about 10 per cent between averages of different runs on different dates. (Better data were obtained close to the source in Series I.) The above estimation of the error does not include the very low concentration zone at the upper edge of the plume which was less than 15 per cent of the maximum ground concentration and very small in its absolute value.

Two series of experiments were conducted. In each series three ambient velocities were used —approximately 9, 12, and 16 ft/s. In Series I, the source was located at the boundary at station 33·5 ft (Fig. 1). Measurements of the concentration were taken at 3, 5, 9, 15 and 21 ft downstream from the source. This set of data covered the entire intermediate zone and part of the transition zone. The mass flux of ammonia per unit width in Series I was $G = 0{\cdot}66$ mg/cm-s. In Series II the source was located at station 15·5 ft. Measurements were taken at 17, 23·5, 35·5 and 43·5 ft downstream from the source, thus covering the final zone. The mass flux of ammonia per unit width in Series II was $G = 0{\cdot}55$ mg/cm-s.

Measurements of the concentration in the transverse direction indicated that the field was approximately two dimensional. Some of Wieghardt's measurements of the mean temperature distribution downstream from a line source of heat located in a wind tunnel floor were also used by the authors. The heat-diffusion data used are from Figs. 11 and 12 of reference 5

Unfortunately, Wieghardt did not report measurements of the velocity profiles and it was necessary to estimate δ using the relationship $\delta = 0{\cdot}37X'[(X'U_{amb})/\nu]^{-1/5}$. The authors have made corrections for the initial laminar section of the boundary layer with $U_{amb} = 5{\cdot}4$ m/s (17·7 ft/s) by assuming a transition at $[(X'U_{amb})/\nu] = 3 \times 10^5$. A turbulence stimulator was used in the case $U_{amb} = 18$ m/s (59 ft/s) and therefore the boundary layer in this case was assumed turbulent from the leading edge.

THE EXPERIMENTAL RESULTS

Introductory remarks

A relative-rate parameter β is defined to assist in dividing the field downstream from the source into zones and in considering the effect of the non-homogeneity of the flow field on the diffusion pattern.

A characteristic length which gives an indication of the rate of change of growth of the boundary layer is

$$L_\delta = \frac{\delta}{(\mathrm{d}\delta/\mathrm{d}x)}. \tag{1}$$

A similar length can be defined to express the diffusion process. If λ is a characteristic height of the region contaminated by tracer matter (hereafter referred to as the plume) then,

$$L_\lambda = \frac{\lambda}{(\mathrm{d}\lambda/\mathrm{d}x)}. \tag{2}$$

The ratio

$$\beta = \frac{L_\lambda}{L_\delta} \tag{3}$$

is a measure of the relative rates of growth of the plume and the momentum boundary layer and will thus indicate how important is the non-homogeneity of the boundary layer in the diffusion process. The value of β near the gas source is determined by the distance of the gas source from the origin of the boundary layer which is assumed to start upstream of the source; however, near the source β will always be small and it will increase with the distance downstream from the source. Whenever the plume and the boundary layer attain a similar rate of growth β becomes constant. Since the vertical-velocity component v is related to the rate of change of the boundary-layer thickness, β, as shown later in equation (14) will also indicate the relative importance of transfer by mean vertical velocity.

Description of the diffusion pattern

Examination of the experimental results indicates that the effect of the non-homogeneity of the field on the diffusion is not uniform and suggests a division of the field into a series of four zones. Other considerations which support such a division of the field will be mentioned later. A description of the diffusion pattern becomes clear and simple by using zones. Approximate limits of the various zone in terms of x/δ_{av} as defined in Fig. 7 are suggested.

(1) *The initial zone.* Very large velocity and concentration gradients made it impossible to obtain reliable data close to the source with the available equipment. It is, however, possible that the laminar sublayer and the large longitudinal gradients which are negligible further downstream will affect the diffusion process in this region. The upper limit of (x/δ_{ave}) was not determined because of difficulty in measuring the concentration profile near the source. Moreover, one expects this limit to be related to some characteristic height of the laminar sublayer rather than to (x/δ_{ave}) alone.

(2) *The intermediate zone.* The diffusing plume, within this zone, is submerged in the boundary layer; but, its thickness is large compared to that of the laminar sublayer. Longitudinal gradients are small compared to vertical gradients and the boundary-layer-type approximation becomes possible. The ratio β is small and the diffusion depends only slightly on the rate of the boundary-layer growth.

The mean-concentration profiles can be described by a dimensionless universal curve

$$\frac{c}{C_{max}} = f(\xi) \tag{4}$$

where

$$\xi = \frac{y}{\lambda} \text{ and } f(1) = 0{\cdot}5$$

as shown in Fig. 5. The function $f(\xi)$ appears to be independent of U_{amb} and δ in this zone and is

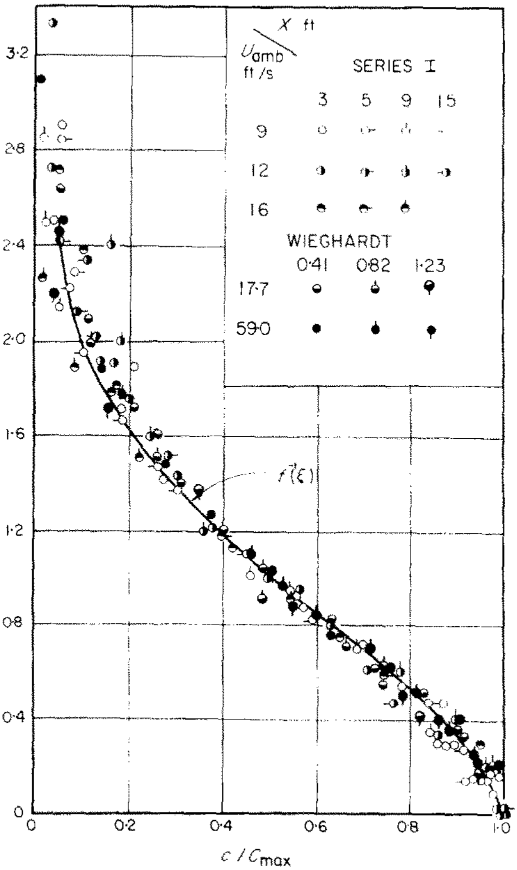

FIG. 5. (c/C_{max}) vs (y/λ) in the intermediate zone.

described in Fig. 5. Variation of λ initially is given by

$$\lambda = 0{\cdot}076\, x^{0{\cdot}8} \tag{5}$$

where x and λ are measured in cm. Slight deviation of the data from equation (5) when $U_{amb} = 59$ ft/s is noted.

The values of C_{max} appear to be inversely proportional to U_{amb}. The initial variation of $C_{max}\, U_{amb}$ (in c.g.s. units) can be approximated by

$$\left.\begin{aligned} C_{max}\, U_{amb} &= 17{\cdot}3\, x^{-0{\cdot}9}, \\ \text{or} \quad & \\ C_{max}\, U_{amb} &= 26{\cdot}2\, G\, x^{-0{\cdot}9} \end{aligned}\right\} \tag{6}$$

The variation of β and (λ/δ) is given in Fig. 7. The curve shown for β is not a universal curve since β also depends upon the location of the source relative to the boundary-layer origin. A decrease in the rate of growth of (λ/δ) is noted beyond $(x/\delta_{av}) = 18$ where (λ/δ) is about 0·39. At the same time, the shape of the concentration profiles changes from that described by $f(\xi)$ (see Fig. 11). The value of $(x/\delta_{av}) = 18$, therefore, can be taken as an approximate upper limit of this zone.

(3) *The transition zone.* The effect of the mild mixing processes in the ambient air is to decrease the rate of growth of the diffusing plume and to gradually change the shape of the concentration profile.

Within the zone, $18 < (x/\delta_{av}) < 60$, β increases to unity. Downstream of $(x/\delta_{av}) = 60$, (λ/δ) remains constant at 0·64.

(4) *The final zone.* Diffusion of matter beyond the boundary layer into the ambient air is controlled by the molecular action and the turbulent fluctuations in the ambient air, similar to the control of the diffusion of momentum. The final zone starts at approximately $(x/\delta_{av}) = 60$. The limited length of the test section did not permit measurements in all the zones for the same position of the gas source. Measurements in the final zone were taken during different flow conditions—Series II—in which the source was moved upstream a distance of 18 ft as shown in Fig. 1.

The concentration profiles within this zone can be described by

$$\frac{c}{C_{max}} = f\left(\frac{y}{\delta}\right). \tag{7}$$

In Fig. 10, the empirically determined form of F is shown. The ground concentration C_{max} shown in Fig. 9 can be approximated by

$$C_{max} \propto (U_{amb}\, \delta)^{-1},$$

or

$$C_{max} = \frac{(G/0{\cdot}55)}{U_{amb}\, \delta} \tag{8}$$

when c.g.s. units are used.

ANALYTICAL EXAMINATION OF THE EXPERIMENTAL RESULTS

The conservation of mass for the two-dimensional case is expressed by the equation

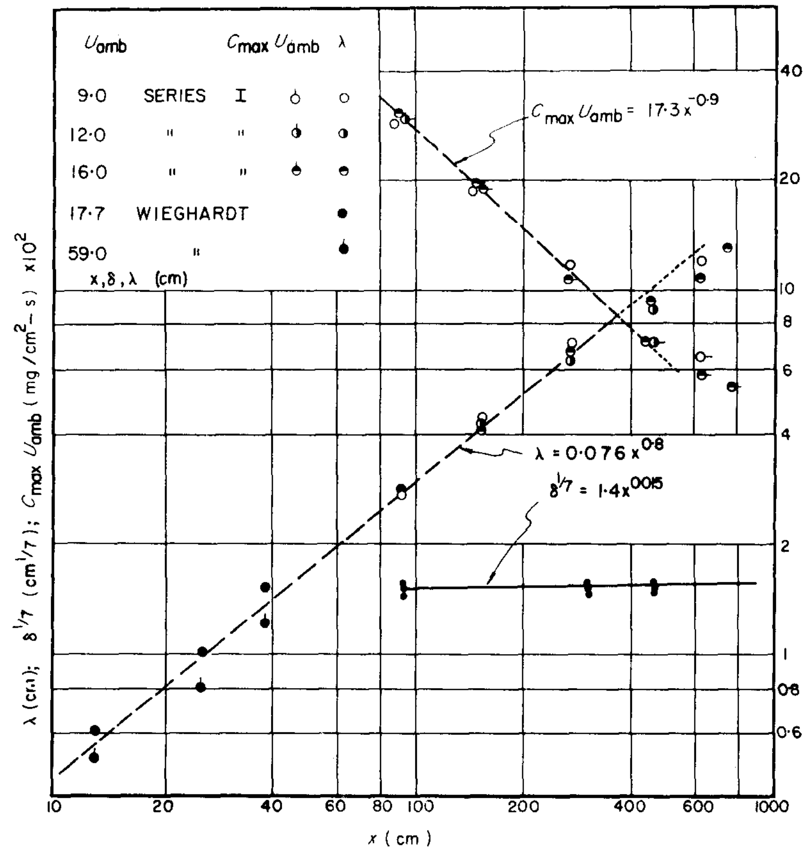

FIG. 6. Variation of λ and C_{max} U_{amb} in the intermediate zone.

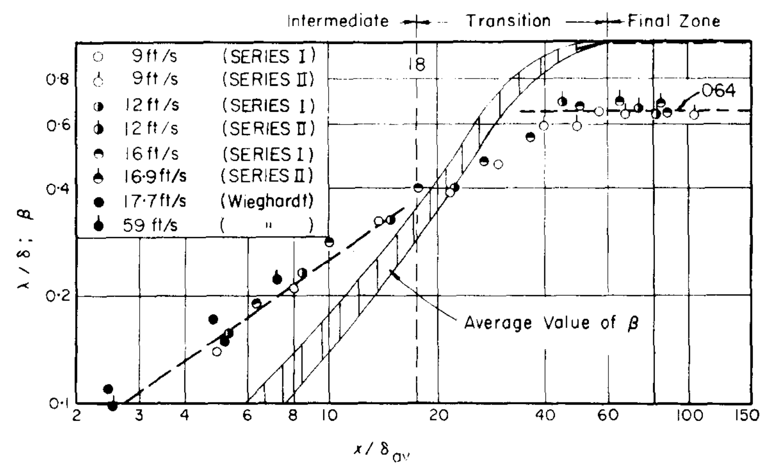

FIG. 7. The variation of (λ/δ) and β with (x/δ_{av}).

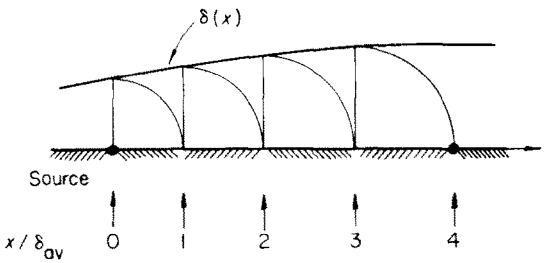

FIG. 8. Determination of (x/δ_{av}).

$$u\frac{\partial c}{\partial x}+v\frac{\partial c}{\partial y}=\frac{\partial}{\partial y}\left(k\frac{\partial c}{\partial y}-\overline{v'c'}\right)+\frac{\partial}{\partial x}\left(k\frac{\partial c}{\partial x}-\overline{u'c'}\right). \quad (9)$$

Except near the source, a boundary-layer-type approximation of equation (9) becomes possible and gives:

$$u\frac{\partial c}{\partial x}+v\frac{\partial c}{\partial y}=\frac{\partial}{\partial y}\left(k\frac{\partial c}{\partial y}-\overline{v'c'}\right). \quad (10)$$

Integration of equation (10) may be achieved by using the distribution functions obtained in the experiments. The variation of $\overline{v'c'}$ and $\epsilon=-[(\overline{v'c'})/(\partial c/\partial y)]$ can thus be examined.

The intermediate zone

Consider the following mean velocity and concentration fields (Figs. 2 and 5):

$$c=C_{\max}f(\xi) \quad (4)$$

where

$$\xi=\frac{y}{\lambda} \quad \text{and} \quad f(1)=0{\cdot}5,$$

and

$$u=U_{\text{amb}}\,\eta^{1/n},\ (n\sim 7). \quad (11)$$

Since c vanishes as y becomes large

$$\int_0^\infty cu\,dy=G \quad (12)$$

where G is a constant of the diffusion field and is equal to the flux of the diffusing quantity per unit time and width. It follows that

$$C_{\max}U_{\text{amb}}\lambda\left(\frac{\lambda}{\delta}\right)^{1/n}\int_0^\infty \xi^{1/n}f(\xi)\,d\xi=G$$

and according to equation (4)

$$c=\frac{G}{\int_0^\infty \xi^{1/n}f(\xi)\,d\xi}\cdot\frac{\delta^{1/n}f(\xi)}{\lambda^{(n+1)/n}\,U_{\text{amb}}}. \quad (13)$$

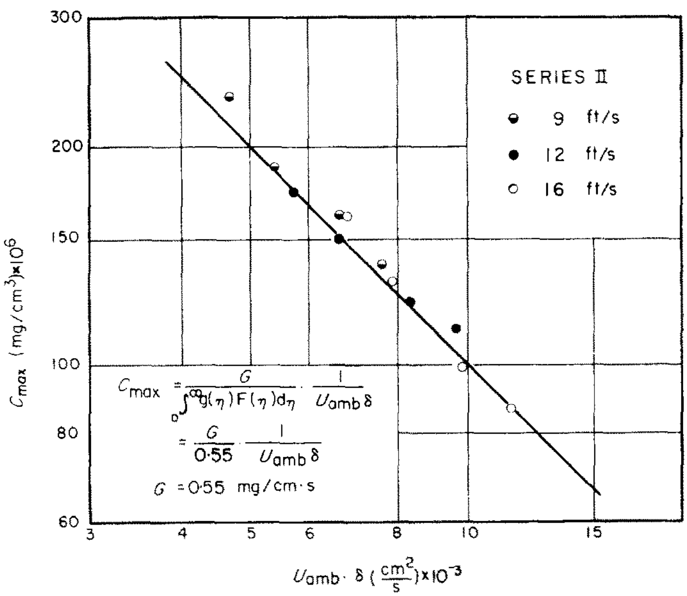

FIG. 9. The variation of $C_{\max}$ with $U_{\text{amb}}\,\delta$ in the final zone.

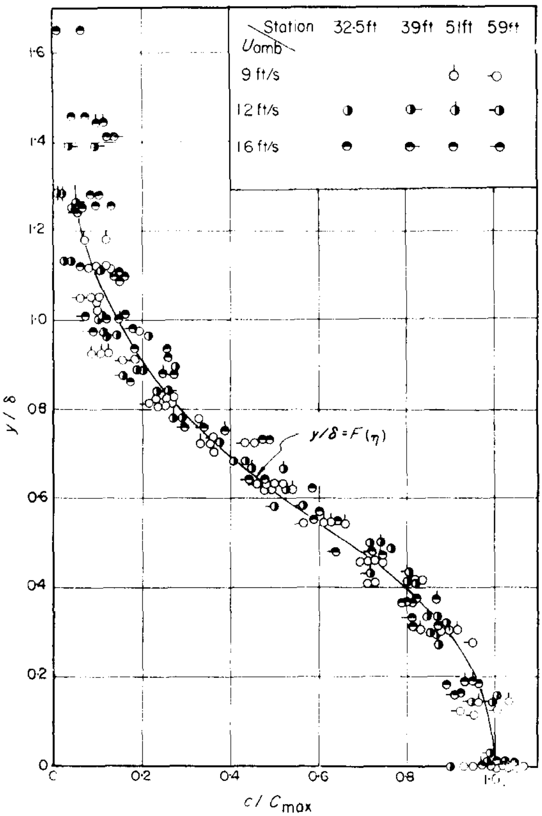

FIG. 10. (c/C_{max}) vs (y/δ) in the final zone.

The value of $\int_0^\infty \xi^{1/n} f(\xi)\,d\xi$ was evaluated from the data with $n = 7$ and is approximately equal to 0·98. The mutual variation of the parameters $\delta^{1/n}$, λ and $C_{max}\,U_{amb}$ shown in Fig. 6 is consistent with equation (13). Equation (10) can be integrated by using equations (11) and (13).* The integration gives:

$$k\frac{\partial c}{\partial y} - \overline{v'c'} = -\frac{G}{\lambda}\frac{d\lambda}{dx}F(\xi)\,[1 - \beta/(n+1)] = -\frac{G}{\lambda}\frac{d\lambda}{dx}S_\lambda(\xi, \beta) \qquad (14)$$

where

$$F(\xi) = \frac{\xi\,(n + 1/n) f(\xi)}{\int\limits_0^\infty \xi^{1/n} f(\xi)\,d\xi}$$

and β is the ratio defined in equation (3).

* Originally the integration was made with $\delta^{1/n}$ taken as a constant. However, Dr. M. V. Morkovin in the process of review indicated that the integration could be achieved just as simply without this assumption.

As seen in Fig. 7, within the intermediate zone β varies from 0·1 to 0·4 with the relative contribution resulting from the factor $\beta/(n+1)$ being of the order of 1/20; therefore, the last term of equation (14) will be neglected in making comparisons of $\overline{v'c'}$ in the different zones. This means that $S_\lambda(\xi, 0)$ will be used as a reference function. If $\overline{v'c'}$ is separated according to the Fickian model $\overline{v'c'} = -\epsilon\,(\partial c/\partial y)$ one obtains

$$k + \epsilon = -\lambda\frac{d\lambda}{dx}U_{amb}\left(\frac{\lambda}{\delta}\right)^{1/n}\frac{S_\lambda(\xi, \beta)}{f'(\xi)}\int\limits_0^\infty \xi^{1/n} f(\xi)\,d\xi. \qquad (15)$$

The function $f(\xi)$ can be estimated from Fig. 5; however, the evaluation of $f'(\xi)$ from the same figure is not reliable. Using the experimentally determined $f(\xi)$, $S_\lambda(\xi, 0)$ was determined by graphical methods and is plotted in Fig. 12.

Although $f'(\xi)$ was not evaluated, one can estimate ϵ at the beginning of the intermediate zone by using the following values:

$$\lambda = 3\text{ cm},\quad \frac{d\lambda}{dx} = 0{\cdot}024,\quad \int\limits_0^\infty \xi^{1/n} f(\xi)\,d\xi = 0{\cdot}98,$$

$$\left(\frac{\lambda}{\delta}\right)^{1/7} = 0{\cdot}75,\quad f'(\xi) = -0{\cdot}6\text{ (maximum)},\quad f(\xi) = 0{\cdot}25,$$

$$U_{amb} = 260\text{ cm/s},\quad k = 0{\cdot}23\text{ cm}^2/\text{s}.$$

Substituting into equation (15) one gets

$$\epsilon \simeq 5{\cdot}5\text{ cm}^2/\text{s} \gg k.$$

Since, according to equation (15), ϵ increases with x, it seems justified to neglect k in the entire intermediate zone except near the boundary. Neglecting the molecular-diffusivity term one gets

$$\epsilon = -\lambda\frac{d\lambda}{dx}\left(\frac{\lambda}{\delta}\right)^{1/n}U_{amb}\frac{S_\lambda(\xi, \beta)}{f'(\xi)}\int\limits_0^\infty \xi^{1/n} f(\xi)\,d\xi. \qquad (16)$$

$$\overline{v'c'} = \frac{G}{\lambda}\frac{d\lambda}{dx}S_\lambda(\xi, \beta). \qquad (17)$$

The final zone

Similar integration in the final zone is possible even without approximating the velocity profile in a power law. Using the distribution functions

$$\frac{c}{C_{\max}} = F(\eta) \tag{7}$$

and

$$\frac{u}{U_{\text{amb}}} = g(\eta) \tag{18}$$

where

$$\eta = \frac{y}{\delta} \text{ and } g(1) = 0{\cdot}99$$

in the integral equation of mass conservation, the following expression for $C_{\max}$ is found:

$$C_{\max} = \frac{G}{\int_0^\infty g(\eta)\, F(\eta)\, d\eta} \, \frac{1}{U_{\text{amb}}\, \delta}. \tag{19}$$

Integration of equation (10), neglecting the molecular term, gives:

$$\overline{v'c'} = \frac{G}{\delta} \frac{d\delta}{dx} S_\delta(\eta)$$

where

$$S_\delta(\eta) = \frac{F(\eta) \int_0^\eta g(z)\, dz}{\int_0^\infty F(\eta)\, g(\eta)\, d\eta} \tag{20}$$

and

$$\epsilon = U_{\text{amb}}\, \delta \frac{d\delta}{dx} E(\eta) \tag{21}$$

where

$$-E(\eta) = \frac{F(\eta)}{F'(\eta)} \int_0^\eta g(a)\, da.$$

It is instructive to derive similar expressions for $\overline{v'c'}$ and ϵ in the case of diffusion in homogeneous turbulence [9] where

$$c = \frac{G}{u - (\sqrt{\pi}/2)\, \sigma} \exp - \left\{\frac{y^2}{2\sigma^2}\right\}$$

and the mass-conservation equation is

$$u \frac{\partial c}{\partial x} = -\frac{\partial}{\partial y} \overline{v'c'} = \frac{\partial}{\partial y}\left(\epsilon \frac{\partial c}{\partial y}\right).$$

Integrating the mass-conservation equation one gets

$$\overline{v'c'} = -\left(\sqrt{\frac{2}{\pi}}\right) \frac{G}{\sigma} \frac{d\sigma}{dx} S_\sigma \left(\frac{y}{\sigma}\right). \tag{22}$$

where

$$S_\sigma\left(\frac{y}{\sigma}\right) = \frac{y}{\sigma} \exp - \left(\frac{y^2}{2\sigma^2}\right)$$

and

$$\epsilon = U_{\text{amb}}\, \sigma \frac{d\sigma}{dx}. \tag{23}$$

In general $\sigma\, (d\sigma/dx)$ is a function of x, however, when x is very large and $\sigma \propto x^{1/2}$, ϵ becomes a constant—the limiting case in homogeneous turbulence. The structure of equations (16), (21) and (23) is similar but unfortunately within the

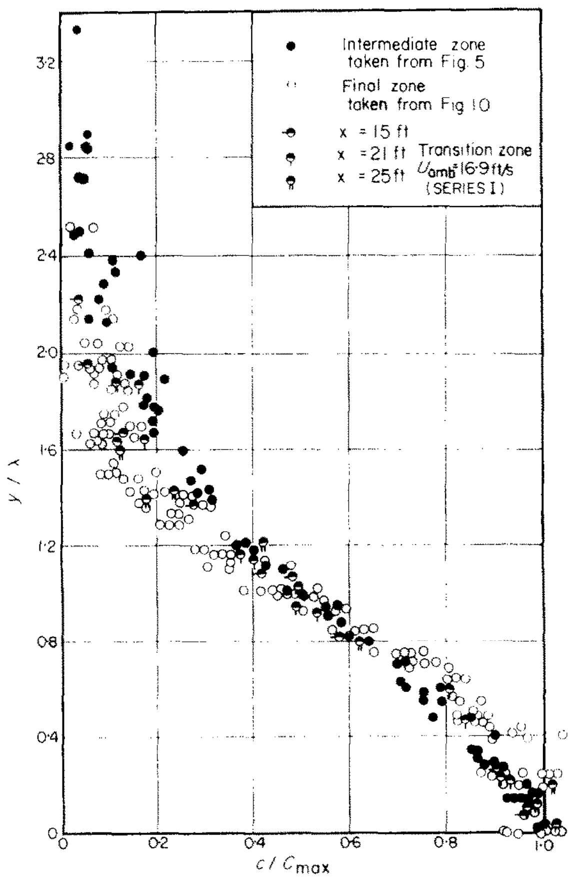

FIG. 11. Comparison of $(c/C_{\max})$ vs (y/λ) in the various zones.

boundary layer ϵ does not become independent of either the vertical or the horizontal co-ordinate. Comparing S_σ with S_λ and S_δ (Fig. 12) we find that the distribution of this dimensionless function is very similar except that the value of S_δ drops off faster as one approaches the edge

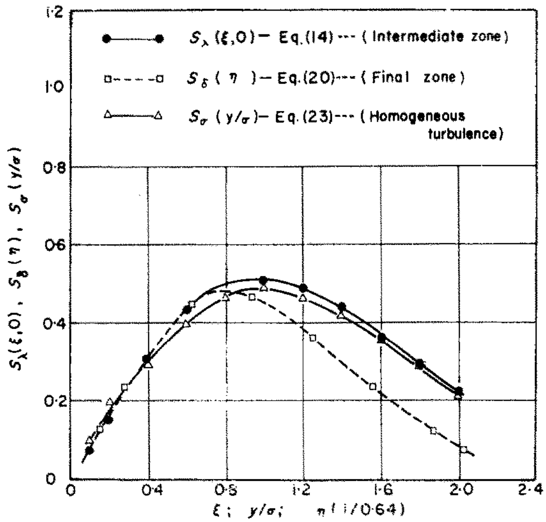

FIG. 12. Dimensionless functions related to $\overline{v'c'}$

of the plume. The decrease of S_δ together with the increase of $\partial c/\partial y$ (Fig. 11) is due to the reduction of the turbulent transport at the outer edge of the layer.

DISCUSSION

The intermediate zone

Within the intermediate zone, where the diffusing plume is totally submerged in the boundary layer, the rate of growth of the vertical dimension of the plume is large compared to the rate of growth of the boundary layer itself and thus β is small (0·1–0·4). Accordingly, the diffusion pattern is affected little by boundary-layer changes within the zone.

Equation (5), determined from Fig. 6, indicates that the vertical scale of the plume is independent of the ambient velocity. It implies that the agents of the flow which control the vertical diffusion within the boundary layer are proportional to the ambient velocity in such a way that the vertical transfer of the mass is approximately proportional to the convection of mass by the longitudinal velocity.

Now, the formulation of the results in the form $\lambda = 0{\cdot}076\, x^{0\cdot 8}$ and the above conclusion should be regarded as an approximation since they do not take into consideration the size of the boundary layer and the changes which take place in the velocity field. The small value of $\beta/(n+1)$ in this region indicates that the *rate* of change of the boundary layer is not important. The deviation of the data obtained at the velocity $U_{\text{amb}} = 59$ ft/s from the above formula is therefore a result of the different rate of growth and thickness of the boundary layer near the source rather than experimental scatter.

The same arguments hold with regard to Wieghardt's formulation of his data. Wieghardt approximated his finding by the expression

$$\frac{\theta}{\theta_{\max}} = \exp\left\{-\left[\frac{y}{F_1(x)}\right]^{\alpha}\right\}$$

where θ is the temperature increase, and found that $F_1(x)$, which can be regarded as a measure of the plume size similar to λ, varies as

$$F_1(x) = 0{\cdot}55\, x\left(\frac{U_{\text{amb}}\, x}{\nu}\right)^{-1/5} = 0{\cdot}55\, x^{0\cdot 8}\left(\frac{U_{\text{amb}}}{\nu}\right)^{-1/5}.$$

This formulation implies that the pattern of diffusion is completely independent of the thickness of the boundary layer and that the diffusion pattern will be the same if the source is placed close to or far away from the leading edge. In his attempts to formulate the data in this manner, Wieghardt found it necessary to vary α from 1·64 for $U_{\text{amb}} = 17{\cdot}7$ ft/s to 2·0 for $U_{\text{amb}} = 59$ ft/s.

It appears to the authors that a more adequate formulation of the data is in terms of the parameter (λ/δ) and (x/δ_{av}) as shown in Fig. 7. Such a formulation accounts for the non-homogeneity of the space and the thickness of the boundary layer at each section. One can see in Fig. 7 that data of Wieghardt with $U_{\text{amb}} = 59$ ft/s agree better with the other data when formulated in this manner.

Equation (16) exhibits the shortcomings of the Fickian model and the concept of an eddy diffusivity. One hopes to find that ϵ is a function of the flow field and that its value at a point can be specified as a function independent of the position of the source. However, the form of equation (16) indicates that this cannot be so. Recalling that the intermediate zone can be regarded as an approximate model for atmospheric diffusion from a ground source in the absence of buoyancy forces, one concludes that a description of the ability of the atmosphere to diffuse matter in terms of an ϵ varying only with height is incomplete and misleading.

It should be remarked that an initial dependence of ϵ on the distance from the source is expected. As in the case of diffusion in homogeneous turbulence such a dependence would probably last for a distance of the order of the Lagrangian integral scale. Direct measurements of the Lagrangian integral scale are not available. It is shown, however, that a time-delayed, dimensionless velocity correlation can maintain large values for a longitudinal distance of four boundary-layer thicknesses [10]. Measurements by Baldwin and Mickelsen [11] in a pipe flow show that the space-time correlation coefficients have a magnitude of about 0·2 at separation distances of 16 pipe radii. Reference 11 shows that these Eulerian space-time correlations might be approximately equal to the Lagrangian time correlation. It is therefore possible to assume that the Lagrangian integral scale of the boundary layer will be of the order of 10 boundary-layer thicknesses.

Another interesting result is the similarity of the distribution of $\overline{v'c'}$ in the boundary layer and in homogeneous turbulence as shown by equations (17) and (22) and Fig. 12. In both cases, $\overline{v'c'}$ is inversely proportional to the characteristic length scale of the diffusing plume and the dimensionless distribution is very similar.

The final zone

Some of the features of the diffusion, such as the dependence of $\overline{v'c'}$ and of c on (G/U_{amb}) are the same throughout the diffusion field. The major difference between the intermediate zone and the final zone is that the characteristics of the diffusion field are independent of the position of the source in the final zone, as expressed by equations (7) and (8).

Once such relations are established, it is possible to relate parameters like $\overline{v'c'}$ and ϵ to the velocity field as shown in equations (19) and (20). It is also possible to relate ϵ to other parameters like $\epsilon_m = -[(\overline{u'v'})/(\partial u/\partial y)]$, however, the various expressions are related and none of them expresses a true relation between the phenomena and its causes.

It should be realized that for this range of Reynolds numbers the developing boundary layer is not self preserving [12], which means that the characteristics of the diffusion will change together with the boundary layer and any similarity will be limited to a certain range of Reynolds numbers. The changes will be mild for large Reynolds numbers; however, the Reynolds number is undoubtedly a parameter to which the diffusion process is related.

The second parameter upon which the diffusion process depends as suggested by the dimensionless equations is the Schmidt number (ν/k). Although the importance of the molecular diffusivity in determining the spatial distribution of the diffusing scalar is fundamental, one realizes that it is mainly the turbulent motion which causes the rapid dispersion of matter in the turbulent boundary layer. It is expected, therefore, that even for large Schmidt numbers the matter will quickly diffuse and "fill" the turbulent boundary layer and that further growth of the plume will be similar to the growth of the boundary layer.

If the value of k is increased, it is clear that the diffusion rate of mass near the upper edge of the boundary layer will be amplified and that the plume size will increase more rapidly. It remains to be asked whether, for very small Schmidt numbers, the plume will increase indefinitely beyond the boundary layer and a similarity will not be established. Additional experimental studies are needed to obtain an answer to this question.

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Résumé—La diffusion d'une quantité scalaire (gaz ammoniac) dans une couche limite turbulente bidimensionnelle à partir d'une source linéaire permanente est étudiée. En utilisant une longue veine d'essai de section carrée 183 × 183 cm, l'épaisseur de la couche limite variait de 12,5 à 27,5 cm pour des distances en aval de 91 cm à 13,26 m avec des vitesses de l'air de 2,75 à 4,88 m/s. Les mesures des concentrations moyennes du gaz ammoniac dans l'air sont rapportées, analysées et comparées avec quelques mesures de transport de chaleur dans des conditions similaires. La formulation des résultats prend en considération l'inhomogénéité transversale du champ des vitesses et également divise le champ de diffusion aval en quatre zones. Les mesures du champ de vitesses moyennes et du champ de concentrations moyennes permet de calculer le flux de masse dans la direction verticale à partir de l'équation de conservation de la masse. L'emploi d'un coefficient de diffusion turbulente pour décrire les processus de diffusion turbulente est discuté et on montre que pour une longue distance en aval de la source, un tel coefficient ne peut pas être relié aux variables locales d'Euler du champ de vitesses.

Zusammenfasseng—Die Diffusion von Ammoniak von einer gleichmässigen linearen Quelle innerhalb einer zweidimensionalen turbulenten Grenzschicht wird untersucht. In einer langen, quadratischen (183 × 183 cm) Versuchsstrecke variierte die Grenzschichtdicke von 12 bis 28 cm in Entfernungen von 90 bis 1330 cm stromabwärts bei Luftgeschwindigkeiten von 2,7 bis 4,9 m/s. Messergebnisse für mittlere Ammoniakkonzentrationen in Luft wurden analysiert und mit den wenigen Messungen des Wärmeüberganges unter ähnlichen Verhältnissen verglichen. Die Formulierung der Ergebnisse berücksichtigt die Inhomogenität des Geschwindigkeitsfeldes in Querrichtung und führt zur Aufteilung des stromabwärtsgelegenen Diffusionsfeldes in vier Zonen. Die Messungen des mittleren Geschwindigkeitsfeldes und des mittleren Konzentrationsfeldes ergeben mit Hilfe des Gesetzes von der Erhaltung der Masse den Massenstrom in senkrechter Richtung. Die Verwendung eines turbulenten Austauschkoeffizienten zur Beschreibung der turbulenten Diffusion wird diskutiert und es wird gezeigt, dass ein derartiger Koeffizient für eine grosse Strecke stromabwärts von der Quelle nicht mit den örtlichen Euler'schen Variablen des Geschwindigkeitsfeldes verbunden werden kann.

Аннотация—Исследуется диффузия скалярной величины (аммиачный газ) от стационарного линейного источника в плоском турбулентном пограничном слое. С помощью длинной 6 × 6 футов квадратной трубы (рабочей части) толщина пограничного слоя изменяется от 5 до 11 дюймов на расстоянии от 3 до 43,5 дюймов вниз по потоку при скорости движения воздуха от 9 до 16 фт/сек. Приводятся, анализируются и сравниваются измерения средних концентраций аммиака в воздухе с некоторыми измерениями переноса тепла в аналогичных условиях. При формулировке результатов во внимание принимается поперечная неоднородность поля скоростей, а диффузионное поле вниз по потоку делится на четыре зоны. Измерение поля средней скорости и поля средней концентрации позволяет рассчитать поток массы в вертикальном направлении по уравнению сохранения массы. Обсуждается использование коэффициента вихревой диффузии для описания процессов турбулентной диффузии и показано, что для большого расстояния вниз по потоку от источника такой коэффициент нельзя отнести к локальным эйлеровым переменным для поля скоростей.